

Nuclear Reactor Theory

Neutrons and Fission

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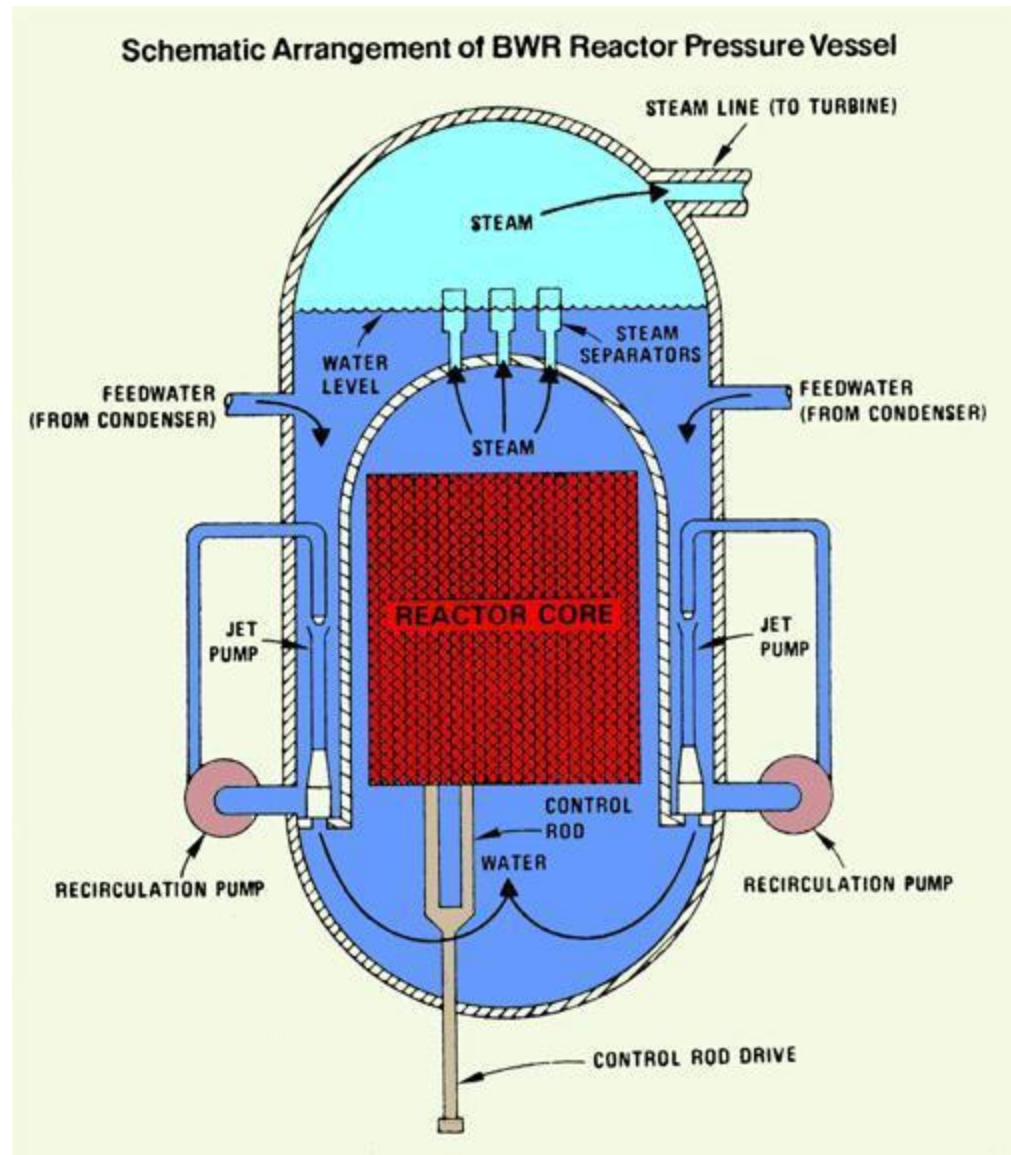
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What is a Nuclear Plant?

- A nuclear plant is a power plant that uses nuclear energy as a heat source to create electricity through classic thermodynamic means.
- Basically, if you take out the nuclear core from the reactor, you will have a steam powered electric generation plant as all other components are the same. Only the heating water part is different as compared to coal or natural gas power plants.

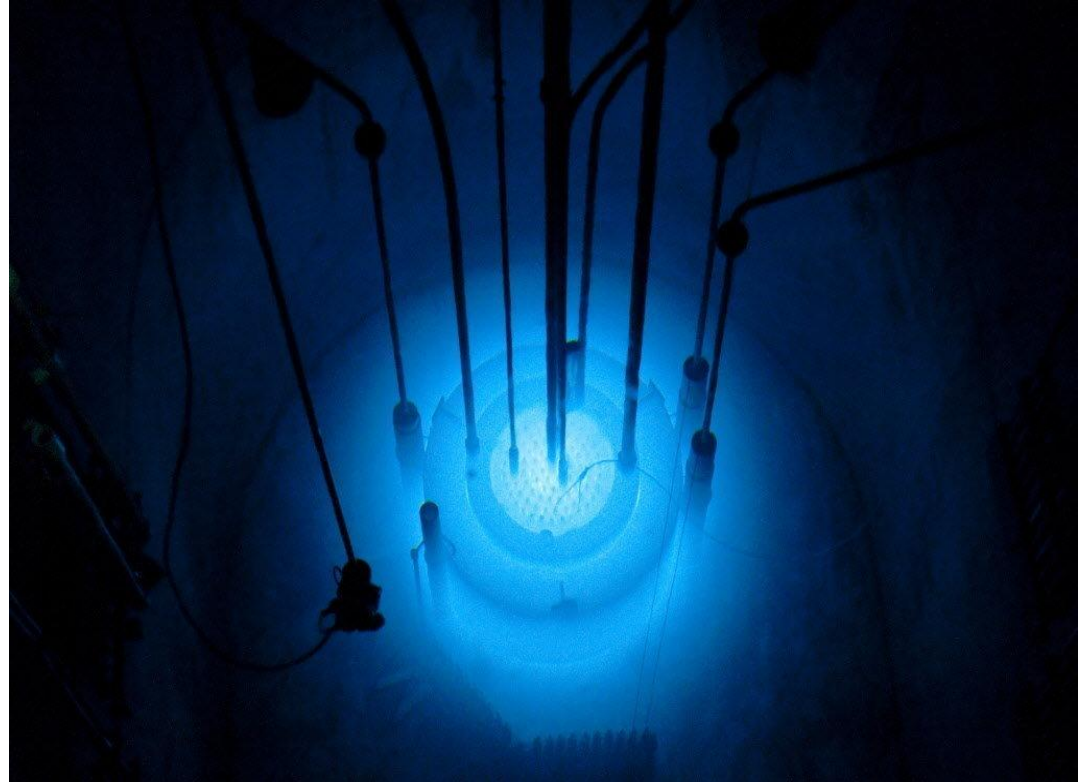
What is a Nuclear Reactor?

- Nuclear Reactor is the part of the Nuclear power plant that is responsible for producing the heat energy through fission reaction and nuclear fuels
- In essence, through various mechanisms, the nuclear reactor is shielded from the rest of the power plants.



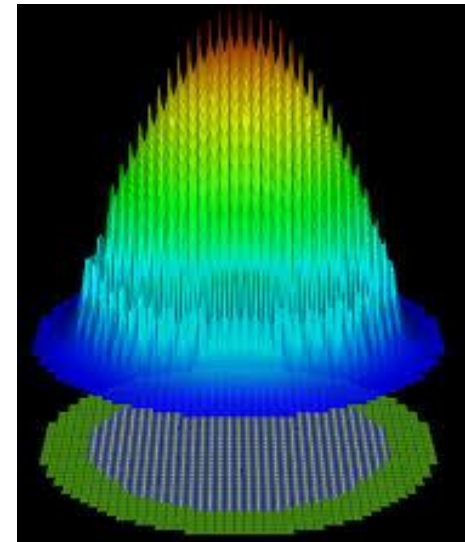
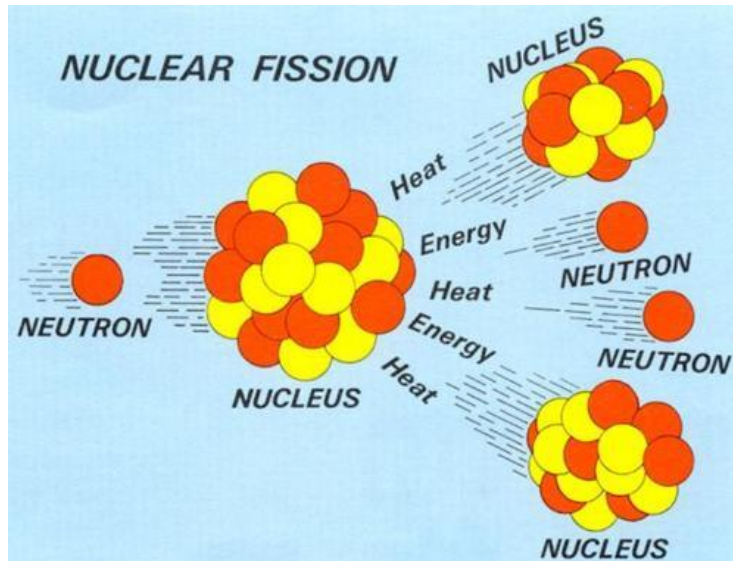
What is the Nuclear Core?

- Nuclear Core is the part of the nuclear reactor that contains the Nuclear Fission reaction that takes place.
- In the nuclear core, due to the fission reaction and due to the converted kinetic energy; thousands of degrees of temperature can be easily reached.
- In the nuclear core, the nuclear fuels, as well as the neutrons which initiate the fission reaction resides. Moreover, the end products of the fission reaction are also found in the nuclear core.



Fission Reaction in the Nuclear Core

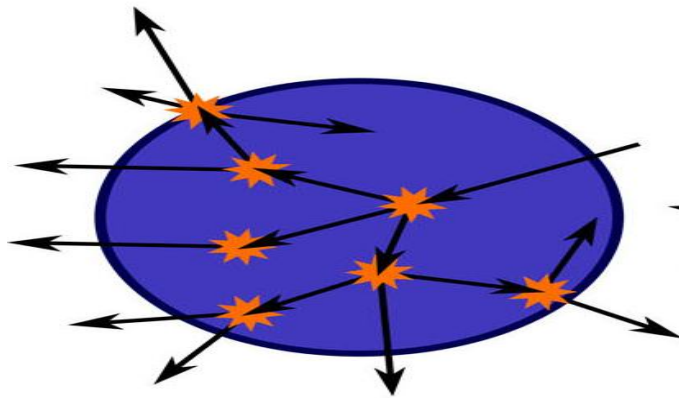
- The main event that takes place in the nuclear core is the fission reaction. In fission, the nuclear fuel is bombarded by neutrons and as a result, the neutrons split the nuclear fuel molecules.
- In the end, new atoms are formed and some neutrons are also formed.



Critical Mass

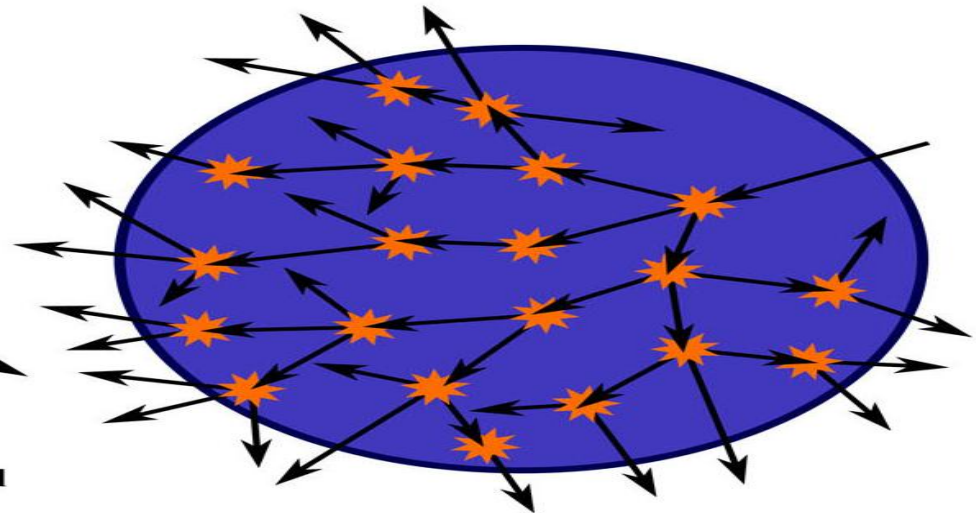
- To achieve a self sustaining reaction, a critical mass of uranium needs to be present. For example if there is only one atom of Uranium 235 it will undergo fission and nothing else will happen as there is no further fuel.
- Hence any amount of nuclear fuel that is less than the amount required for a self sustaining reaction would be called sub-critical.
- **Critical mass** is the threshold amount of mass of nuclear fuel required for self-sustaining reaction.

Sub-critical mass



The sphere of fissile material is too small to allow the chain reaction to become self-sustaining, as neutrons generated by fissions can too easily escape.

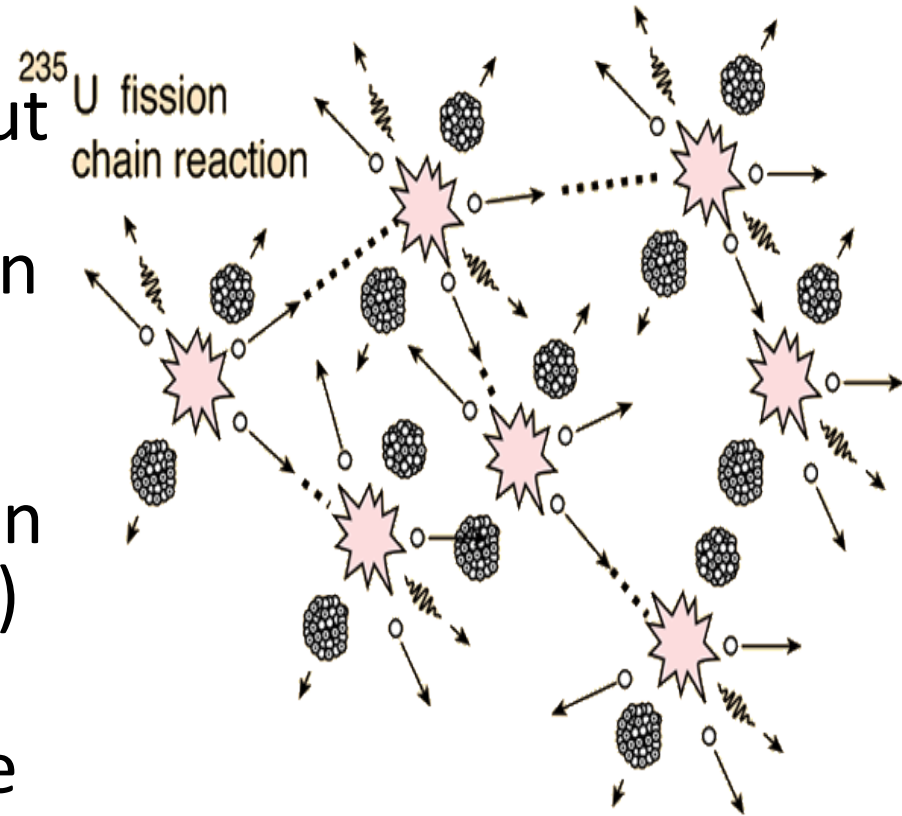
Critical mass



By increasing the mass of the sphere to a critical mass, the reaction can become self-sustaining

Role of the Neutrons in Fission Reaction

- Role of the neutrons is the most important thing, as it is the neutrons which initiate the nuclear reaction. Without the availability of the neutrons, the fission reaction won't take place.
- For fission to happen, neutrons must be at a certain temperature (kinetic energy) level.
- For the fission reaction to be self-sustaining, new neutrons must be continuously formed in the fission reaction



Neutron Energy and Scattering

- Neutrons given off in fission have an energy average of 1 Mev. This energy must be reduced with collisions with the moderator nuclei.
- Since the good moderators fall amongst the lightest elements, the slowing down process is caused almost entirely by elastic scattering or billiard ball collisions between the neutron and the target nuclei, where the energy is conserved.

Cross Sections

- The extent to which neutrons interact with nuclei is described in quantities known as **cross section** σ . It is described in unit of barns.
- If there are n neutrons per cm^3 and v is speed of neutrons then intensity is: **$I = nv$**
- We can also describe number of collisions per second where we have the cross section with the formula for number of collisions per second:

$$\sigma I N A X$$

- The factor NAX is the total number of nuclei in the target. σI is the number of collisions per second with single nucleus. A is cross section area and X is thickness. N you have to get from tables for that atom

Problem 1

- A beam of 1-MeV neutrons of intensity 5×10^8 neutrons/cm² – sec strikes a thin Carbon 12 target. The area of the target is 0.5 cm² and 0.05 cm thick. The beam has a cross sectional area of 0.1 cm². At 1 MeV, the total cross section of C12 is 2.6 barns. At what rate do interactions take place in the target? What is the probability that a neutron in the beam will have a collision in the target?

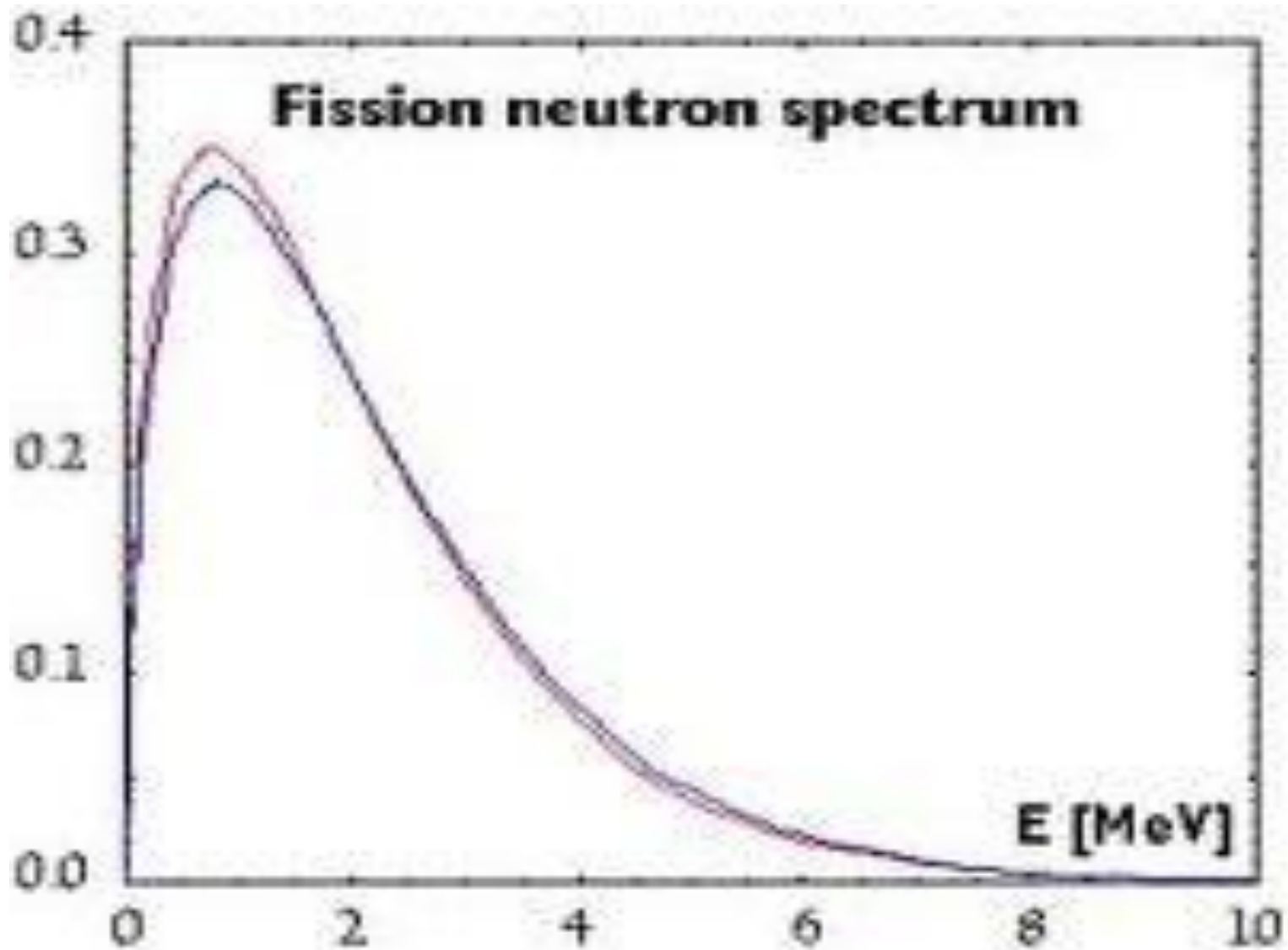
- From tables $N = 0.080 \times 10^{24}$ for carbon

$$\sigma_{\text{TOTAL}} = (2.6 \times 10^{-24}) \times (5 \times 10^8) \times (0.080 \times 10^{24}) \times (0.1) \times 0.05$$
$$= \mathbf{5.2 \times 10^5 \text{ interactions per second}}$$

- In 1 sec, $IA = (5 \times 10^8) \times 0.1 = 5 \times 10^7$ neutrons strike the target and of these 5.2×10^5 interact. The probability that a neutron interacts in the target would be
- $(5.2 \times 10^5) / (5 \times 10^7) = 1.04 \times 10^{-2} = 0.0104$

Hence 1 neutron in 100 has a collision and interacting.

Neutron Cross Sections

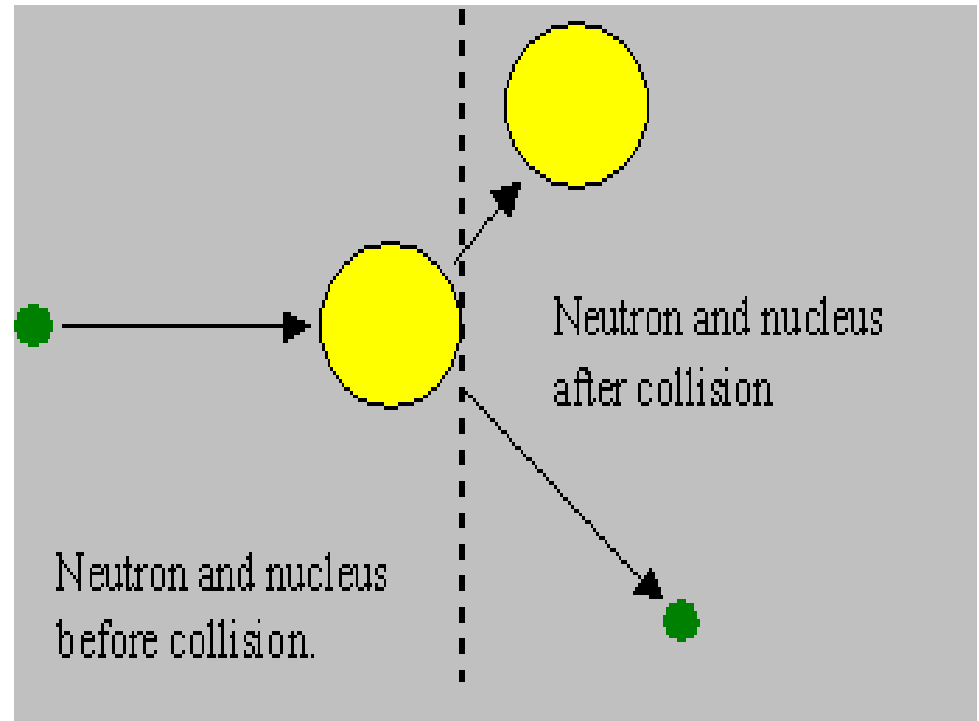


Neutron Scattering

- In collisions with a given moderator, a neutron always loses the same fraction of energy it had before the collision.
- When the energy of the neutron is reduced until it is the same order of magnitude as the thermal energy of the moderator, the neutron will be as likely to gain as to lose energy from a collision and it will simply remain in the thermal region until it is captured.

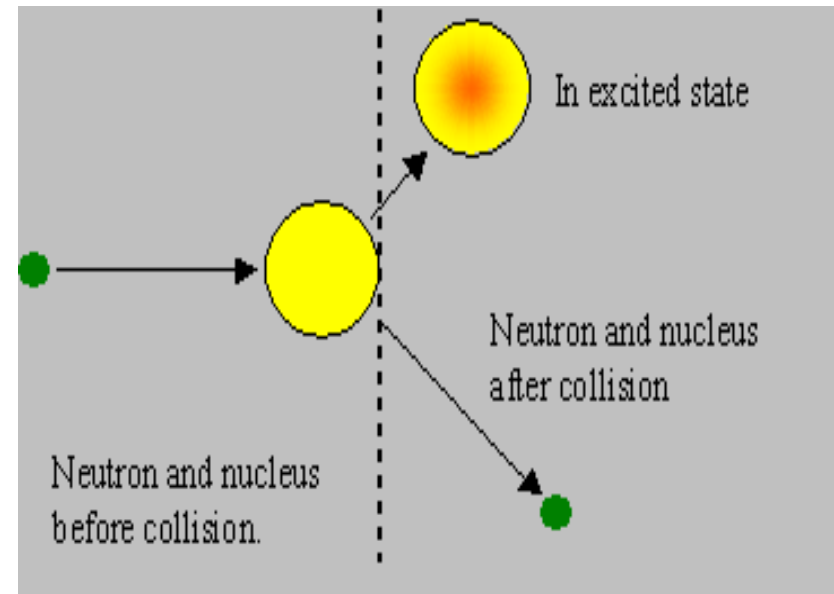
Neutron Elastic Scattering

- The neutron and the nuclide collide and share a part of their kinetic energies. They rebound with speeds different from the original speeds, such that the 'total kinetic energy' before and after the collision remains the same. If the nucleus is stationary before collision, it will gain energy from the neutron and start moving, and the neutron gets slowed down due to loss of kinetic energy. However, the residual nucleus is not excited but is in its ground state.



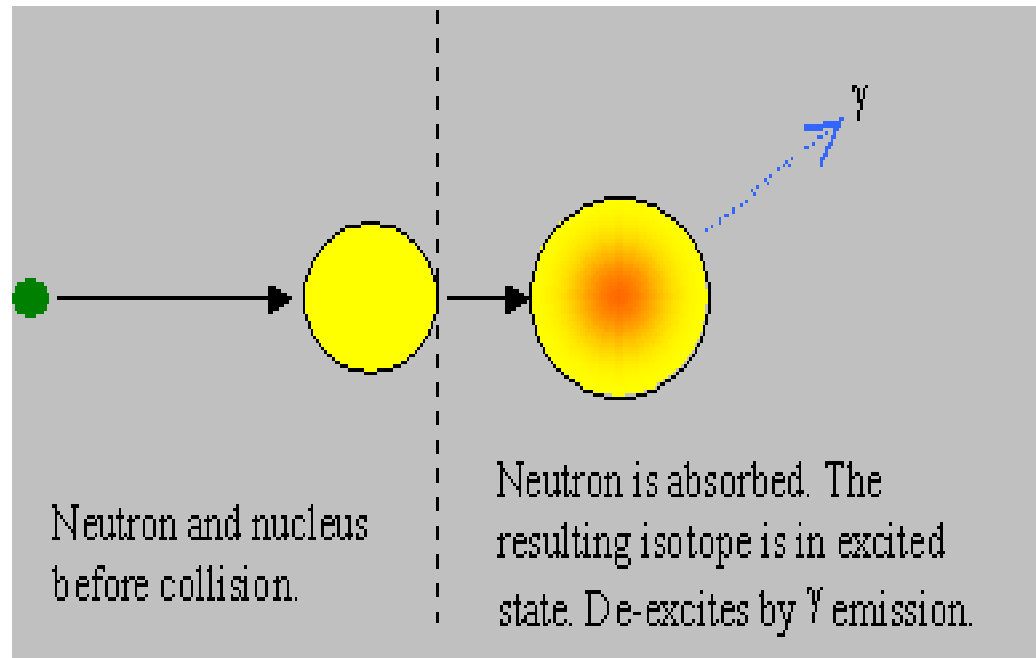
Neutron Inelastic Scattering

- The neutron and the nuclide collide and rebound with speeds different from the original speeds, but the rebounding nuclide is left in an excited energy state. Hence the 'total kinetic energy' after the collision is less than that before the collision, and this difference accounts for the energy of excitation.
- The excited nucleus subsequently de-excites by emitting gamma radiation. Though the probability of inelastic scattering is generally lower than elastic, the energy loss to the neutron is higher in an inelastic collision



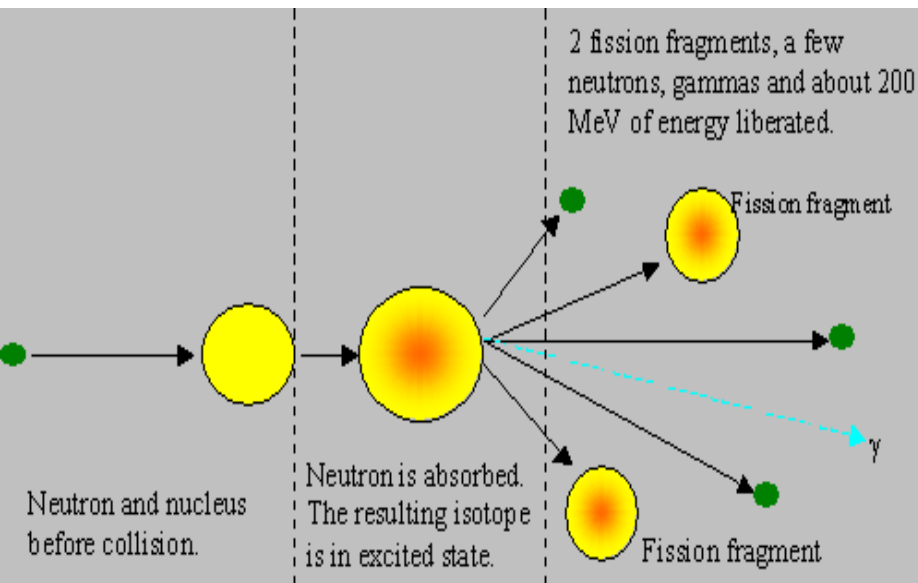
Neutron Capture

- The neutron is absorbed by the target nucleus to form the next higher isotope (of mass $A+1$), in an excited state of energy. The new isotope de-excites by emitting gamma rays. The neutron is thus lost in this reaction.



Fission and Neutron Energy

- The neutrons produced in fission are fast, **with an average energy of 2 MeV.**
- It must be noted that the fission fragments themselves are in excited state, and they de-excite generally by β , γ and neutron emissions
- About 80 % of the energy released in fission is carried away by the fission products (and the rest by the other particles), which in turn transfer the energy to the surroundings, making the energy recoverable. Some energy is carried away by particles known as neutrinos, which are chargeless and light, do not interact with any material, and hence their energy is not recoverable



Fission



Total Cross Section

- Hence, neutron will undergo interaction as per its cross section for each interaction. Thus, these interaction cross sections are namely for elastic scattering, inelastic scattering, radiative capture, and also fission by fission cross section.

$$\sigma_t = \sigma_e + \sigma_i + \sigma_\gamma + \sigma_f$$

- The total scattering cross section is the sum of the elastic and inelastic cross section and total cross section would be total of scattering and absorption.

$$\sigma_s = \sigma_e + \sigma_i$$

$$\sigma_t = \sigma_s + \sigma_a$$

Problem 2

- When 0.0253 eV neutrons interact with Uranium 235, only radiative capture and fission can occur as absorption reaction. The cross sections for these reactions are 99 b and 582 b respectively. What is the relative possibility for fission to occur under these conditions?
- Since only radiative capture and fission can take place, then the probability will be:

$$\frac{\sigma_f}{\sigma_\gamma + \sigma_f}$$

- Hence $582/(582+99) = 85.5\%$

Fission and Energy Transfer

- The neutrons emitted during fission are called **prompt neutrons**, and those emitted by the fragments after a delay are called **delayed neutrons**. Similarly, prompt and delayed gammas are also emitted. Roughly 202 MeV of energy is emitted.

Kinetic energy of fission fragments	170 MeV
Prompt radiations (gammas and neutrons)	12 MeV
Delayed radiations (neutrons and gammas)	12 MeV
Neutrinos (unrecoverable)	8 MeV
Total	202 MeV

Average Loss of Energy in Collisions

- Average loss in the logarithm of the energy in one collision is abbreviated by ξ
- The value of ξ only depends on the mass of the scattering nucleus and it is independent of the initial energy of the neutron.
- The greater the value of ξ , the less will be the number of collisions required to thermalize the neutron.

$$\xi = \frac{2}{A + \frac{2}{3}}$$

Problem 3

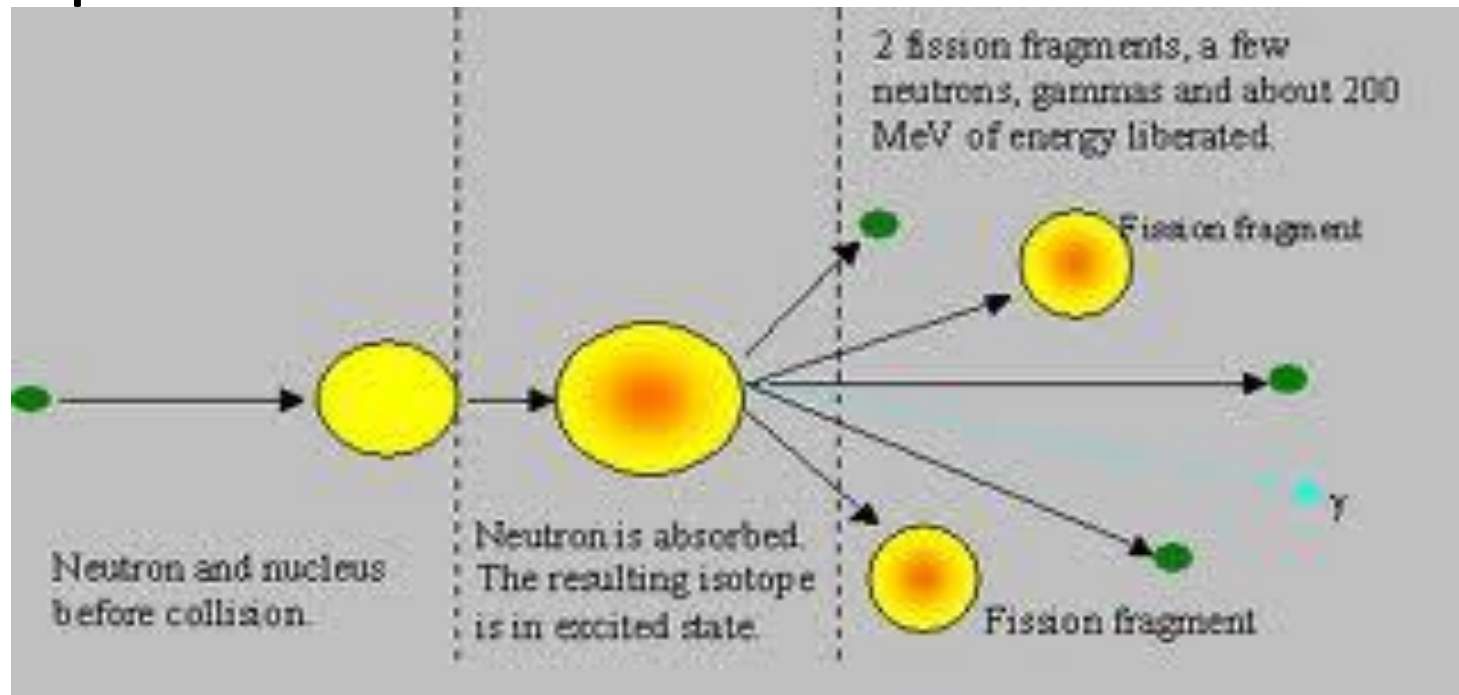
- Calculate the average number of collisions required with a carbon nucleus to reduce the energy of a fast neutron from 2 Mev to 1/30 ev?
- For carbon : $\xi = \frac{2}{12 + \frac{2}{3}} = 0.158$
- Total Reduction of Energy = $\frac{2 \times 10^6}{\frac{1}{30}} = 6 \times 10^7$
- Logarithmic energy reduction = $\ln (6 \times 10^7) = 18$
- Average Number of Collisions = $18 / 0.158 = 114$

Neutrons Must be Kept Under Control

- Hence, neutrons must be kept under control in the nuclear reactor in order to control the nuclear fission reaction.
- Too many neutrons will cause excessive nuclear fission to take place, while too few will cause the reaction to die out.
- Neutrons must be protected from leaving the reactor core and they must be also kept at a thermalized temperature that allows them to initiate fission reaction

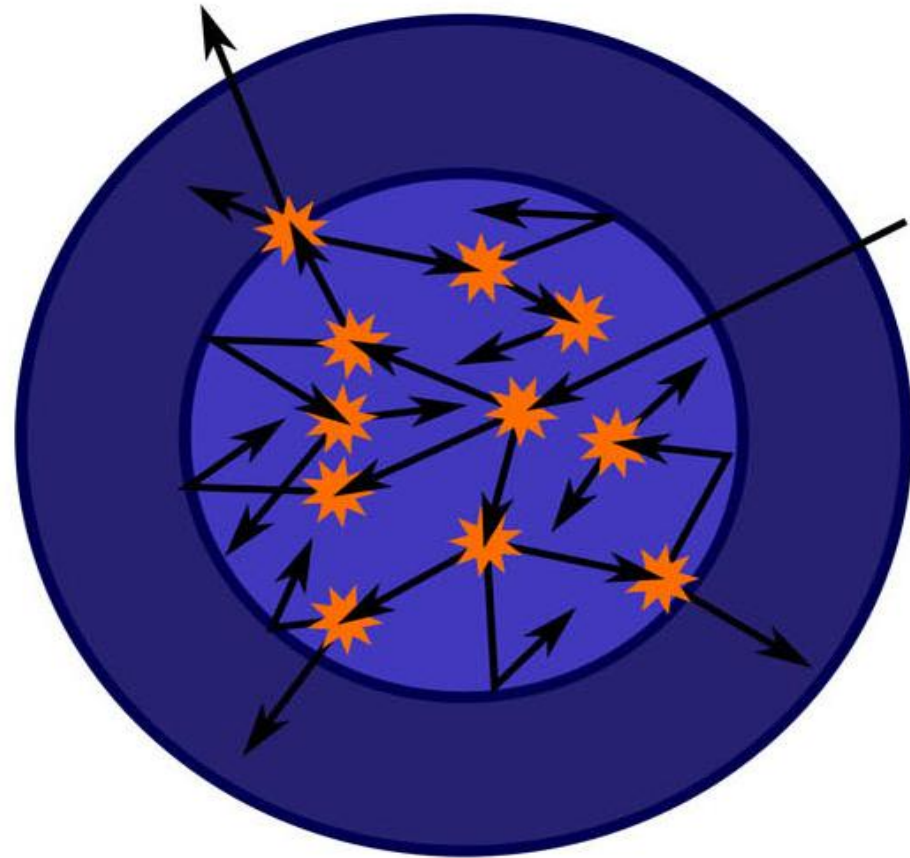
Neutron Moderators

- One of the best moderators is water as water is a heavy molecule and a neutron that hits the water molecule will lower its kinetic energy and become thermalized.
- Some other forms of moderators are carbon graphite.



Neutron Reflectors

- Neutron shielding is also used to reflect the neutrons back in to the nuclear core, so that the neutrons can continue to initiate nuclear fission reaction without leaving the reactor chamber.
- Again water and graphite can be used as neutron reflectors too. Without a good neutron reflector, you will have to keep supplying new neutrons to keep the fission reaction critical.



Surrounding the original sphere with a neutron reflector increases the efficiency of the reactions and also allows the reaction to become self-sustaining

Macroscopic Cross Section

- The product of atom density N and cross section (σ) is called Macroscopic Cross Section.
- Since N has units of cm^{-3} and σ has cm^2 , macroscopic cross section has units of cm^{-1}
- Macroscopic cross section shown as:

$$\Sigma = N\sigma$$

Problem 4

- Take the same values given in Problem 1. Calculate the macroscopic cross section for Carbon 12 at 1 MeV.
- N for Carbon is 0.08×10^{24}
Cross Section is 2.6×10^{-24}
 $\Sigma = (0.08 \times 10^{24}) \times (2.6 \times 10^{-24})$
 $\Sigma = 0.21 \text{ cm}^{-1}$

Thermal Utilization Factor

- The thermal utilization factor (f) is simply the ratio of number of neutrons absorbed by the fuel to the number of neutrons absorbed by all of the processes.

$$f = \frac{N_1(\sigma_a)_1}{N_1(\sigma_a)_1 + N_2(\sigma_a)_2 + \dots} = \frac{(\Sigma_a)_1}{(\Sigma_a)_1 + (\Sigma_a)_2 + \dots}$$

- N_1 = Number of atoms of fuel / cm³
- N_2, N_3 = Number of atoms/cm³ of moderator and coolant
- σ_a = microscopic cross section for the material analyzed
- $(\Sigma_a)_1$ = macroscopic cross section of the material being analyzed

Mean Free Path

- The average distance that a neutron moves between collisions is called the mean free path. It is the amount of distance that neutron moves without any collision

$$\lambda = \frac{1}{\Sigma}$$

Problem 5

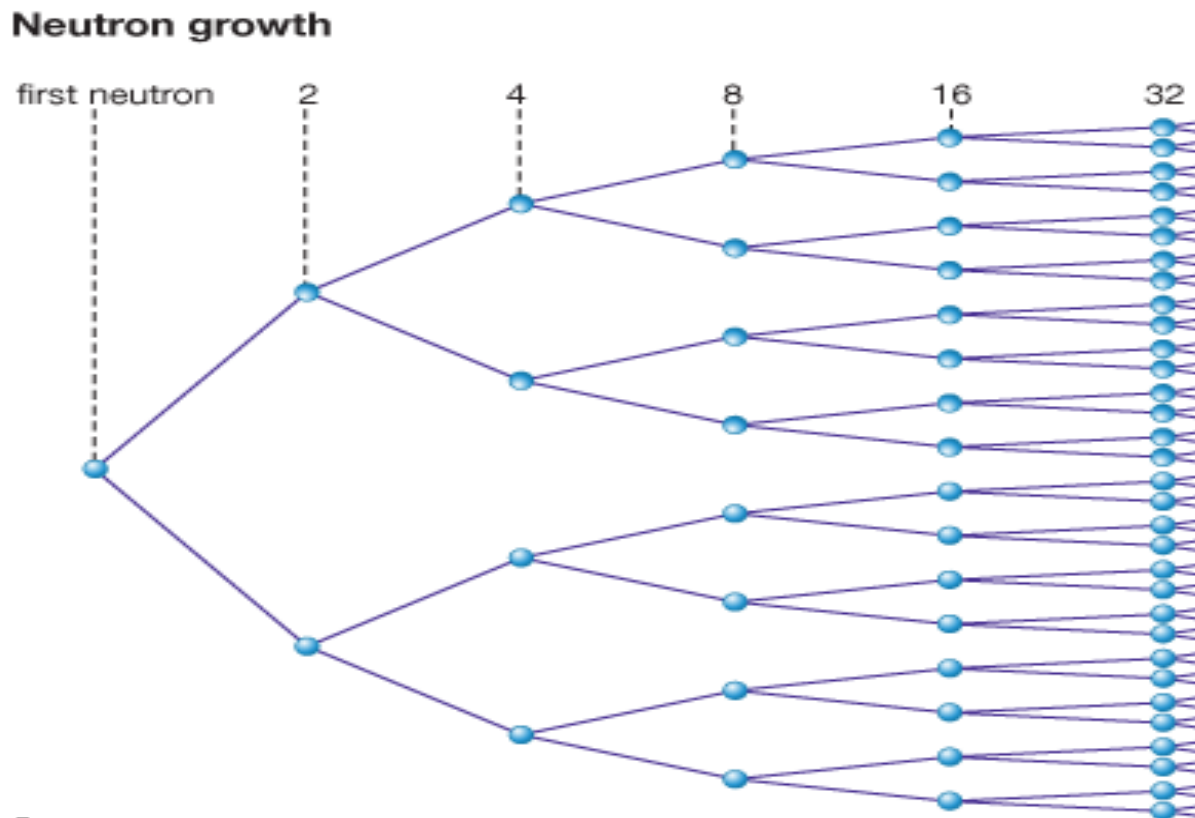
- Calculate the mean free path of 100 keV neutrons in liquid sodium. At this energy, the total cross section of sodium is 3.4 barns.
- The atom density of sodium is 0.0254×10^{24}

$$\Sigma = 0.0254 \times 10^{24} \times 3.4 \times 10^{-24} = 0.0864 \text{ cm}^{-1}$$

$$\lambda = 1/0.0864 = 11.6 \text{ cm}$$

Neutron Growth

- For the fission reaction to be self sustaining, there must be stable neutron growth, otherwise the reaction will die off.



Neutron Multiplication Factor

- The neutrons in the chain reaction will start to multiply in a self sustaining reaction. The amount of multiplication in the fission reaction is called the multiplication factor

$$k = \eta \epsilon p f$$

- η : multiplication constant (depends on the fuel and for uranium it is 1.32)
- ϵ : fast fission factor (probability of fast neutrons undergoing fission) (For uranium it is 1.2, it increases with the diameter of the fuel rods)
- p : moderator to fuel ratio (usually 0.874)

Multiplication Factor and Criticality Constant

- If the value of $k > 1$, then the chain reaction will pick up speed and there will be more fission reactions occurring per second. **Supercritical**
- If the value of $k = 1$, then the reaction will continue self sustaining with the same rate. **Critical**
- If the value of $k < 1$, then the reaction will eventually die out as the number of fission reactions will decrease **Subcritical**

Neutron Population and Power

- The power of a reactor is directly proportional to the neutron population .
- If there are more neutrons in the system, more fission will take place producing more energy.
- **When a reactor is starting up, the neutron population is increased slowly in a controlled manner, so that more neutrons are produced than are lost, and the nuclear reactor becomes supercritical.** This allows the neutron population to increase and more power to be produced. When the desired power level is achieved, the nuclear reactor is placed into a critical configuration to keep the neutron population and power constant.
- **Finally, during shutdown, the reactor is placed in a subcritical configuration so that the neutron population and power decreases.**
- Therefore, when a reactor is said to have “gone critical,” it actually means it is in a stable configuration producing a constant power.

Criticality is a Balanced State

- In the context of nuclear power, “criticality” indicates that a reactor is operating safely.
- In the balanced state of criticality, fuel rods inside a nuclear reactor are producing and losing a constant number of neutrons, and the nuclear energy system is stable.
- A nuclear reactor is a system that controls this criticality or balance of neutrons.